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Publication date:
1996

Document Version
Publisher's PDF, also known as Version of record

Link to publication from Aalborg University

Citation for published version (APA):
Engelund, S., & Sørensen, J. D. (1996). *Stochastic Models for Chloride-Initiated Corrosion in Reinforced Concrete*. Dept. of Building Technology and Structural Engineering. Structural Reliability Theory Vol. R9608 No. 161

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PAPER NO. 161

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Printed at Aalborg University

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Stochastic Models for Chloride-Initiated Corrosion in Reinforced Concrete

Svend Engelund [1] & John D. Sørensen [2]

Abstract

Corrosion of the reinforcement in concrete structures can lead to a substantial decrease of the load-bearing capacity. One mode of corrosion initiation is when the chloride content around the reinforcement exceeds a threshold value. In the present paper a statistical model is developed by which the chloride content in a reinforced concrete structure can be predicted. The model parameters are estimated on the basis of measurements. The distribution of the time to initiation of corrosion is estimated by FORM/SORM-analysis.

Introduction

The reinforcement in concrete structures is protected from corrosion by a chemical as well as a physical barrier. Due to the strong alcalinity of the pore solution, a microscopic oxide layer is formed on the reinforcement which prevents initiation of corrosion. The reinforcement is further protected by the concrete cover. The oxide layer dissolves and corrosion is initiated when the chloride concentration around the reinforcement exceeds a threshold value.

Stochastic Model

Measurements from existing uncracked structures (cracks smaller than 0.1 mm, see e.g. Tuutti (1982)) with not too low w/c-ratios support the assumption that the chloride concentration, c, at a given time, t, can be considered as the solution to a suitable linear diffusion problem, which can be stated as

$$\left.\begin{array}{rcll} \frac{\partial c(\mathbf{x})}{\partial t} & = & \nabla^T\left(\mathbf{D}\left(\mathbf{x}\right)\nabla c\left(\mathbf{x}\right)\right) & \\ c\left(\mathbf{x}\right) & = & c_s & \mathbf{x}\in\{S_g\} \end{array}\right\} \qquad (1)$$

where $\mathbf{x}$ denotes a point in a Cartesian coordinate system, $\mathbf{D}$ is the constitutive matrix, $\nabla^T = \left[\begin{array}{ccc} \partial/\partial x_1 & \partial/\partial x_2 & \partial/\partial x_3 \end{array}\right]$ is the divergence operator , and $\{S_g\}$ denotes the surface.

The random distribution of aggregates, capillary pores and micro cracks within concrete structures leads to an inherent random spatial fluctuation of the consti-

[1]Ph.D.-student, Aalborg Uni. Sohngaardsholmsvej 57, DK-9000 Aalborg, Denmark

[2]Associate Professor, Aalborg Uni., Sohngaardsholmsvej 57, DK-9000 Aalborg, Denmark

tutive matrix. Since this variability is associated with the micro-structure of the concrete, the correlation length of the field describing these fluctuations must be very small (about 2-5 mm). On the other hand, it has been observed from measurements of carbonation that the constitutive matrix exhibits a random spatial variation with a much larger correlation length of about 200-500 mm . Hergenröder (1992) states that this variability can be caused by the spatial variation of the compression of the fresh concrete. The porosity of the outer layers of a concrete structure is usually different from the porosity of the rest of the structure. Hence, the mean value of the constitutive matrix can be assumed not to be constant within the structure. The following general model of the constitutive matrix is considered

$$\{\mathbf{D}(\mathbf{x})\} = \mathbf{D}_0(\mathbf{x}) + \{\mathbf{D}_1(\mathbf{x})\} + \{\mathbf{D}_2(\mathbf{x})\} \tag{2}$$

where $\mathbf{D}_0(\mathbf{x})$ is a deterministic function of $\mathbf{x}$, $\{\mathbf{D}_1(\mathbf{x})\}$ is a zero mean random matrix with a low scale of fluctuation, and $\{\mathbf{D}_2(\mathbf{x})\}$ is a zero mean random matrix with a higher scale of fluctuation.

The random matrices $\{\mathbf{D}_1(\mathbf{x})\}$ and $\{\mathbf{D}_2(\mathbf{x})\}$ in the general 3-dimensional case consist of nine elements which can all be modelled as stochastic fields. However, usually only a very limited number of measurements from a given structure is available, making it difficult to estimate more than a few parameters with sufficient accuracy. Further, it will be difficult to assure that the constitutive matrix is always positive definite. It will always be necessary to apply a simplified model. The model, in fact, must depend on the available data, see the example below.

The surface concentration is also modelled as a stochastic field, $\{c_s(\mathbf{x})\}$. In general, the mean value of the surface concentration will depend on the position, $\mathbf{x}$. In a similar way, the thickness of the cover will exhibit a random spatial fluctuation. The cover thickness is described by a stochastic field, $\{\delta(\mathbf{x})\}$.

Measurements from existing structures indicate that the critical threshold for initiation of corrosion, c_{cr}, exhibits a substantial variation between different structures. Further, the critical threshold depends on the humidity of the concrete, implying that c_{cr} also exhibits a spatial variation within a given structure. The following model is implemented.

$$\{c_{cr}(\mathbf{x})\} = c_{cr_0} + \{c_{cr_1}(\mathbf{x})\} \tag{3}$$

where c_{cr_0} is a stochastic variable and $\{c_{cr_1}(\mathbf{x})\}$ is a stochastic field.

Parameter Estimation

The available data consists of a number of so-called chloride profiles where the chloride concentration is determined as a function of the distance from the surface. Since each of these measurement series only covers about $50 - 70$ mm, $\{D_2(\mathbf{x})\}$ can be assumed to be constant for each series. Hence, we are able to estimate an outcome of $D_0(\mathbf{x}) + \{D_2(\mathbf{x})\}$, the auto-covariance function of $\{D_1(\mathbf{x})\}$ and an outcome of $\{c_s(\mathbf{x})\}$ for each series, see Engelund et al. (1995). The chloride profiles are usually obtained from points on the structure with a relatively large distance. The parameters determined on the basis of each of these profiles, therefore, can be assumed to be independent. Using Bayesian Statistics it is possible to estimate the overall variability of the fields.

Lifetime Estimation

The reinforcement bars are partitioned into elements. In each element the chloride concentration is assumed to be constant. The probability that corrosion is initiated is equal to the probability that the chloride concentration in an arbitrary element exceeds the critical threshold. This problem can be treated by FORM/SORM analysis. The failure probability can be bracketed by the well-known Ditlevsen bounds and an approximation can be found by the Hohenbichler approximation (see e.g. Thoft-Christensen and Murotsu (1986)).

Example

We now wish to determine the probability that corrosion has been initiated in a 1 m by 1 m area on a bridge pier in a marine environment. The available data consist of 4 chloride profiles obtained at $t = 10$ years and 5 measurements of the cover thickness. It is assumed that the penetration of chloride is unidirectional and that the low-scale random fluctuation of the transport coefficient can be neglected.

It is assumed that $\{D_2(\mathbf{x})\}$, $\{c_s(\mathbf{x})\}$ and $\{c_{cr_1}(\mathbf{x})\}$ can be described by homogenous Gaussian fields. On the basis of the measurements we are able to estimate the mean and standard deviation of $\{D_2(\mathbf{x})\}$ and $\{c_s(\mathbf{x})\}$. It is not possible to estimate the auto-correlation coefficient functions. In the example we apply the following auto-correlation coefficient functions

$$\rho_{c_s c_s}(d) = \exp\left(-\frac{d}{a_1}\right) \quad \rho_{D_2 D_2}(d) = \exp\left(-\frac{d}{a_2}\right) \quad \rho_{c_{cr_1} c_{cr_1}}(d) = \exp\left(-\frac{d}{a_3}\right) \tag{4}$$

where d denotes the distance between two points and $a_1 = a_3 = 0.35$ m and $a_2 = 1.0$ m. For the critical threshold the mean value is assumed to be $c_{cr_0} \sim N(0.1; 0.01)$ and the spatial fluctuation is a Gaussian field with zero mean, standard deviation 0.01 and autocorrelation coefficient function given in eq. (4).

In the 1 m by 1 m area there are three reinforcement bars each of which are partitioned into 10 elements of the length of 0.1 m. The following simple model for the concrete cover thickness is implemented

$$\delta(\mathbf{x}) = \delta_0 + A\cos(2\pi(x_1 + x_2)), \quad x_1, x_2 \in \{S_g\} \tag{5}$$

where δ_0 is a constant, A is a normally distributed stochastic variable and x_1, x_2 denote the coordinates of a point on the surface. The parameters in the model are estimated on the basis of measurements of the cover thickness. All stochastic fields are represented by their values at the midpoint of the elements. The probability that corrosion has been initiated can now be determined. The results are shown in figure 1**a**, where no prior information is taken into account, and in 1**b** where prior information corresponding to 5 additional measurements of both the cover thickness and of the transport coefficient and surface concentration have been taken into account.

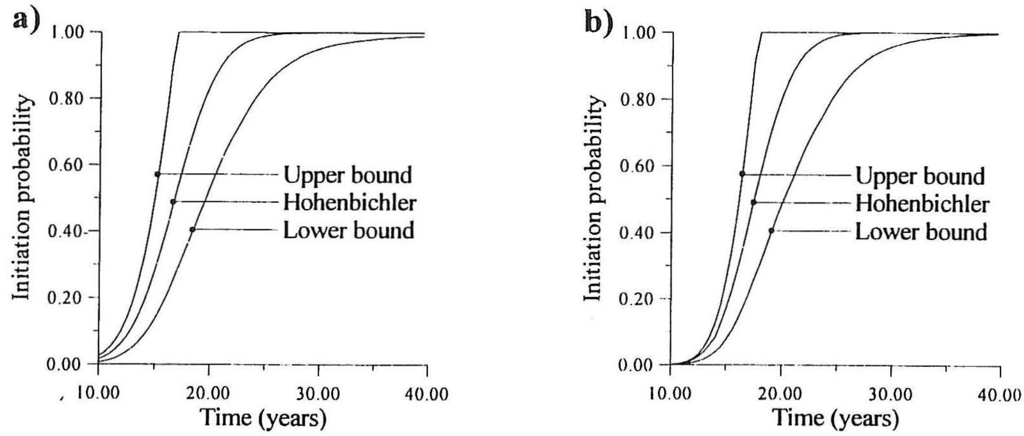

Figure 1: Initiation probability.

Conclusion

The above example is based on measurements on an existing structure and, therefore, the calculated failure probabilities might seem high. However, it must be taken into account that the analysis is based on a limited data and that initiation of corrosion represents no immediate threat to the load-bearing capacity of the bridge. The presented stochastic model can be used in conjunction with planning of measurements of chloride profiles and in planning of maintenance and repair strategies for reinforced concrete structures.

Acknowledgements

The present research was supported by the Danish Technical Council within the research programme on Safety and Reliability.

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Department of Building Technology and Structural Engineering
Aalborg University, Sohngaardsholmsvej 57, DK 9000 Aalborg
Telephone: +45 98 15 85 22 Telefax: +45 98 14 82 43